**Exam Questions – With Solutions**

**Briefly describe the behaviour of the y values for the following, given the behaviour of the x values: y = , as x → –∞**

y → 0

**A parabola with equation has turning point (2, –8) and y-intercept (0, –6). The constants a, b and c are all positive.**

**[a] Determine the values of the positive constants a, b and c.**

y = a(x–2)2 – 8

–6 = a(–2)2 –8 🡪 a = 0.5

y = 0.5(x–2)2 – 8 = 0.5(x+2)(x–6)

a = 0.5, b = 6, c = 2

**[b] The parabola is translated 10 units left and 5 units downwards. Determine the equation of the transformed parabola in the form .**

y = 0.5(x+10–2)2 – 8 – 5

y = 0.5(x+8)2 –13

**An obtuse-angled triangle ABC has a = 36cm, c = 52cm and an area of 748cm2.**

**[a] Sketch a triangle to show this information.**

B

52cm

36cm

A

C

748cm2

**[b] Determine the size of ∠B.**

748 = x 52 x 36 x sin(∠B)

∠B = 53.05° 🡪 But it’s obtuse so ∠B = 180 – 53.05 = 126.95°

**[c] Show that b ≈ 79cm.**

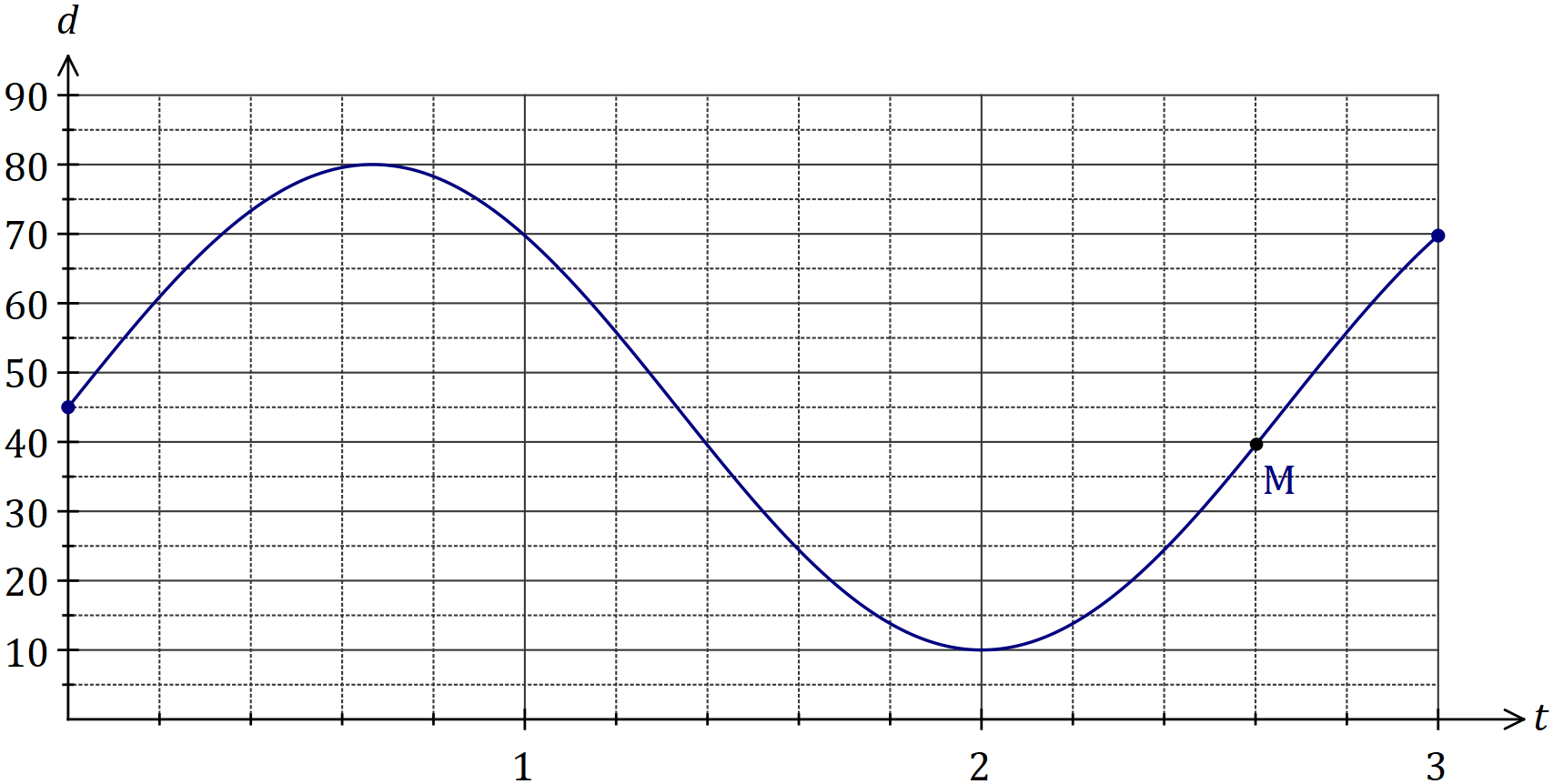
b = = 79.06 ≈ 79

**[d] Show that ∠C ≈ 32°.**

sin(∠C) = 0.526 🡪 ∠C = 31.74° ≈ 32°

**A small weight, attached to the bottom of a spring, oscillated up and down. The distance, d cm, of the weight from the top of the spring after t seconds can be modelled by:**

**[a] Sketch the graph on the axes below for 0 ≤ x ≤ 3.**



**[b] Mark on your graph point M, where the weight is 40cm from the top of the spring and moving downwards.**

Hint: As d gets larger, the spring is moving downwards, so M is as d gets larger.

**[c] Determine**

**(i) The maximum distance of the weight from the top of the spring.**

Maximum distance = maximum value for d = 80cm

**(ii) The time taken for the spring to return to its initial position.**

b =

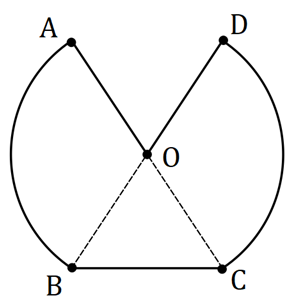
Period = = = 2π x =

Time to return to initial position is half the period so t =0.5 x = = 1.35 seconds

**(iii) The distance moved by the weight between t = 1 and t = 2.**

d(1) – d(2) = 69.75 – 10 = 59.75cm

**In shape OABCD below, ∠AOB = 126° and AC, BD are diameters of the circle with centre O and radius 35cm.**



**[a] Calculate the perimeter of OABCD.**

AB + DC = 2 x 35 x = 153.94cm

BC =

= 31.78cm

AO + DO = 35 x 2 = 70cm

Perimeter = 153.94 + 31.78 + 70 = 255.72cm

**[b] Calculate the area of OABCD.**

AOB + DOC = 2 x x 352 x = 2693.94cm2

BOC = x 352 x sin() = 495.52cm2

Area = 2693.94 + 495.52 = 3189.cm2

**Let a = sin50 and b = cos100.**

**Give your answers to the following in terms of a and/or b.**

**[a] Write an expression for**

**(i) sin130**

sin130 = sin50 = a

**(ii) cos80**

cos80 = – cos100 = –b

**[b] Determine an expression for cos130.**

cos2130 + sin2130 = 1

cos2130 = 1 – a2

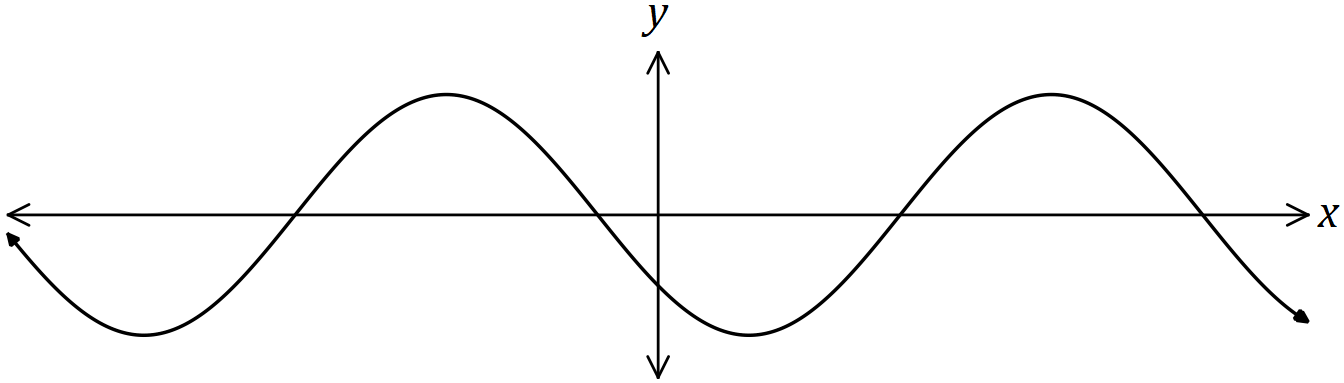
cos130 =

But cos130 is negative 🡪 cos130 =

**[c] Determine an expression for tan130.**

tan130 = =

**The graph of y = f(x) is shown below, where f(x) = is shown below, where f(x) = sin(x+c) and c is a constant.**



**Explain how to obtain the graph of each function below from the graph of f(x), given that a and b are also constants.**

**[a] y = sin(x+a).**

sin(x+c) 🡪 sin(x+a) Subtract c and add a.

sin(x+c–c+a) = sin(x+a)

sin(x+c–(x–a) = sin(x+a)

Translate horizontally by (c–a) units.

**[b] y = cos(x+b).**

sin(x+c) 🡪 cos(x+b)

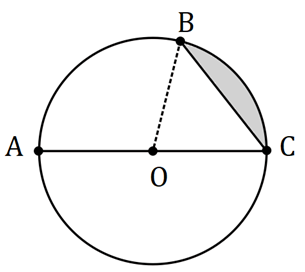
sin(x+c) = cos(x+c) Subtract c and add b.

cos(x+c+ – c+b) = cos(x+b)

cos(x+c–(+c–b) = cos(x+b)

Translate horizontally by (c–b) units.

**The circle shown has centre O and diameter AC of length 50cm. Determine the shaded area given that 2 x ∠AOB = 3 x ∠BOC.**



∠AOB + ∠BOC = 180°

∠BOC = x ∠AOB

CAS simultaneous 🡪 ∠BOC = 72°, ∠AOB = 108°

Area = x 252 x – x 252 x sin() = 95.49cm2

**A sector of a circle has a perimeter of 112cm and an area of 735cm2. Determine the radius of the circle.**

x r2 x θ = 735

r x θ + 2r = 112

CAS simultaneous 🡪 r = 21cm, 35cm

**Line L has equation .**

**[a] Determine the equation of line P that’s perpendicular to L and passes through the point with coordinates (50, 4).**

Line L: y = x + 5

Line P: y = x + c

Substitute (50, 4) 🡪 4 = 40 + c 🡪 c = –36

**[b] Determine the coordinates of the point of intersection of L and P.**

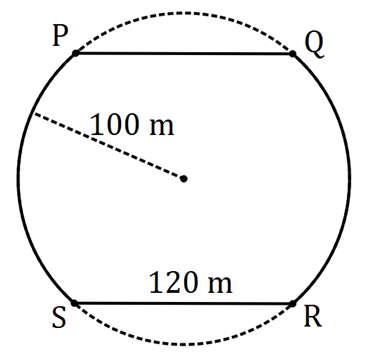
Line P: y = x – 36

Line P = Line L 🡪 x – 36 = x + 5

x = 41 🡪 x = 20

When x = 20, y = –20 🡪 (20, –20)

**A running track has circular ends of radius 100m and two straight, parallel sides and that are both 120m long, as shown below. Determine, to the nearest metre, the total length of the track.**



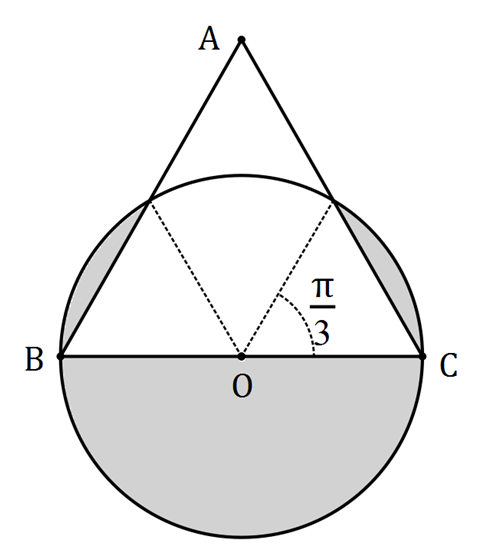
1202 = 1002 + 1002 – 2 x 1002 x cos(x) 🡪 x = 1.287 radians

= 1.855 radians

PS + QR = 2 x 100 x 1.855 = 370.92m

Perimeter = 120 x 2 + 370.92 = 610.92m ≈ 611m

**The diagram shows a circle with centre and diameter , and an equilateral triangle ABC. Determine the exact fraction of the area of the circle that lies outside the triangle.**



Area outside = x π x r2 + 2 x ( x r2 x – x r2 x sin ) = + –

=

Fraction = = x = x =

=

**Line A and Line B in the x-y plane intersect at 90° at the origin. Line A has a slope of . Point (2, –6) is the midpoint of line segment CD which is parallel to Line A. Given that the x-value of C is –1, find the coordinates of point D.**

2 = 🡪 x = 5

= –6 🡪 y2 + y1 = –12

= = = 🡪 y2 – y1 = 2

y2 + y1 = –12

P(D) = (5, –5)

2y2 = –10 🡪 y2 = –5

y2 – y1 = 2

Use gradient: = 🡪 y2 – y1 = 2 🡪 –5 – y1 = 2 🡪 y1 = –7 🡪 P(C) = (–1, 7)

**Complete the square to find the roots of the quadratic function f(x) = 5x2 – 7x + 1.**

5x2–7x + 1 = 5(x2 – x + )

= 5(x2 – x + – + )

= 5(x – )2 –

5(x – )2 =

(x – )2 = x =

x – = = ±

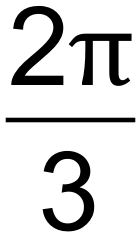
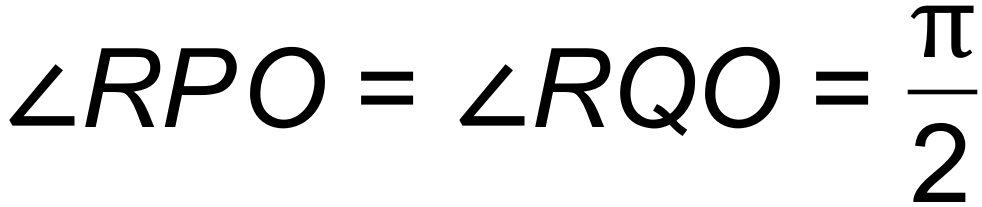
x = ± =

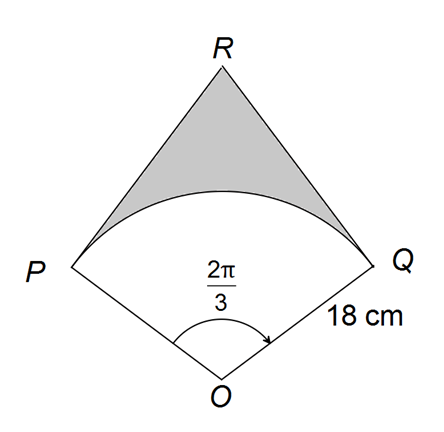
**The current A (amperes), varies inversely to the resistance R (ohms) in an electric circuit. When the resistance is 12 ohms, the current is 0.5 amperes. Determine the effect on R if A is increased by 35%.**

R =

Increased by 35%: R = = 8.89

Effect = x 100 = 25.9% 🡪 Therefore, R decreases by 25.9%

**A sector OPQ of a circle with centre O is drawn below. The radius of the circle is 18 cm and angle POQ is  radians. The tangents to the circle at the points P and Q meet at point R. . Find the exact area of the shaded region.**



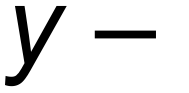
tan() = 🡪 PR = 18tan()

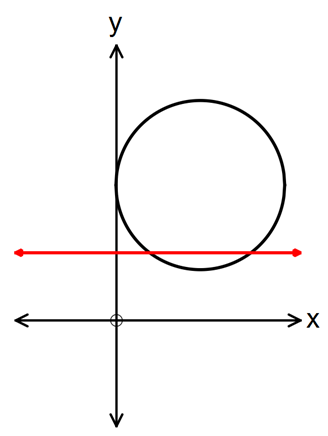
Triangle area = bh = x 18 x 18tan()

Kite area = 2 x x 18 x 18tan() = 18 x 18tan()

Sector area = x 182 x

Shaded area = (18 x 18tan()) – ( x 182 x ) = 324 – 108π = 108(3 – π)

**The circle with centre A(5, 8) touches the axis as shown below.**



**The line y = 4 intersects the circle at point M and N.**

**[a] Determine the length of the chord MN.**

(x – 5)2 + (y – 8)2 = 25 🡪 substitute y = 4 🡪 x = 2, x=8 🡪 6 units

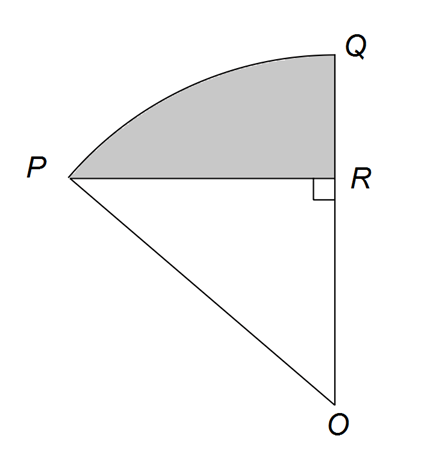
**[b] Find the area of the minor segment formed between MN and the circle.**

62 = 52 + 52 – 2 x 52 x cosθ 🡪 θ = 1.287 radians

x 52 x 1.287 – x 52 x sin(1.287) = 4.0875 units2

**The diagram below has an arc, PQ, of a circle with centre O and radius r.**

**PR is perpendicluar to OQ. Angle POQ =  radians.**



**[a] Show that the area of triangle POR = in terms of r. (Hint: First find expressions for OR and RR in terms of r).**

sin() = 🡪 PR =

cos() = 🡪 OR =

Triangle area = x x =

**[b] If the shaded area is cm2, calculate the value of r.**

Shaded area = sector area – triangle area

= x r2 x –

CAS 🡪 r = 2, r = –2 (reject x = –2)

r = 2

**The perimeter of a sector, with central angle θ radians in a circle of radius r, is 12 cm**

**[a] Express θ in terms of r.**

12 = rθ + 2r

θ = = – 2

**[b] Show that the area of the sector is 6r – r2.**

x r2 x ( – 2) = – r2 = 6r – r2

**[c] Determine the area of the sector if θ = 1.**

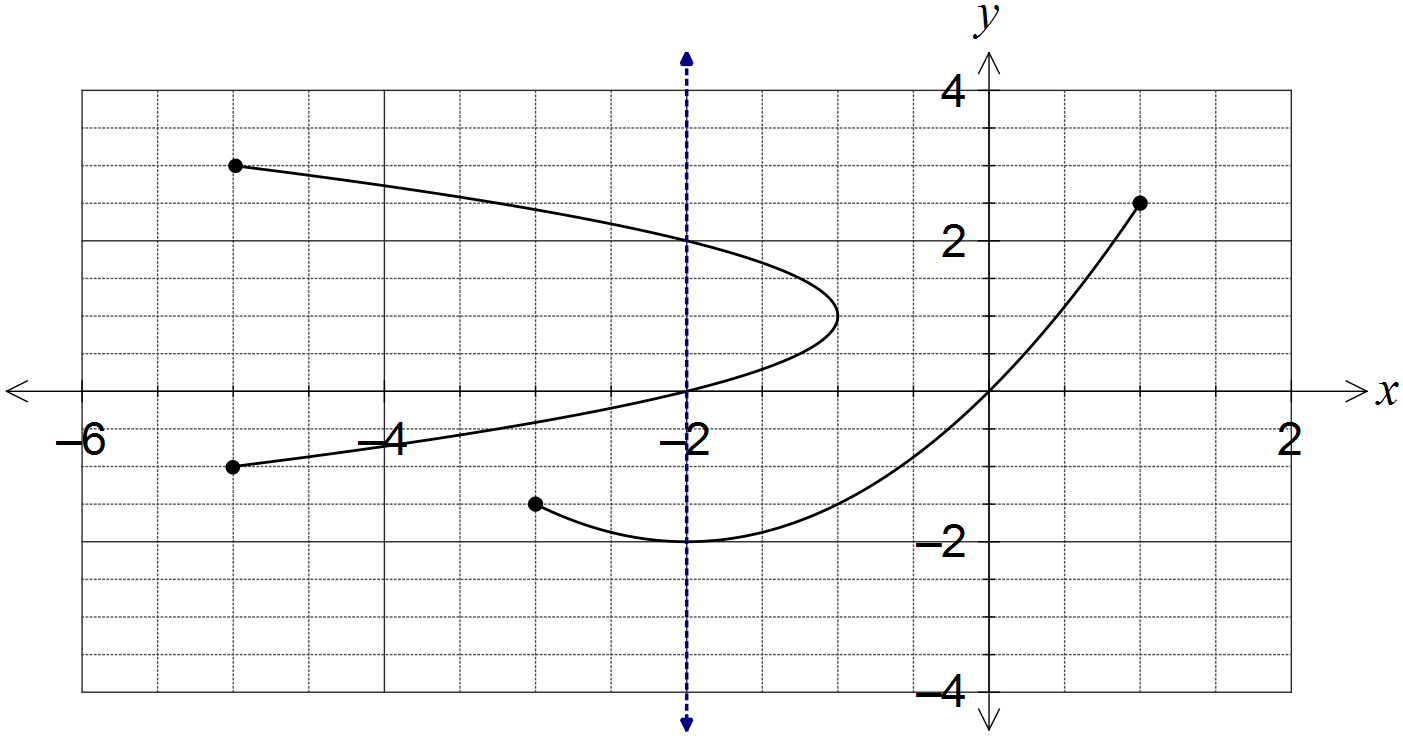
– 2 = 1

12 – 2r = r

–3r = –12

r = 4

x 42 x 1 = 8cm2



**The relation can be expressed in the form y2 = ax + by – 2. Determine the values of the constants a and b.**

When x = 0, y = 0, 2

When y = 0 🡪 02 = –2a + b(0) – 2 🡪 a = –1

When y = 2 🡪 22 = –2(–1) + 2b – 2 🡪 b = 2

**In triangle ABC, ∠BAC, AC = 18.4 cm and BC = 15 cm.**

**Determine the largest possible area and smallest possible perimeter of this triangle.**

B

15

18.4 cm

50°

C

A

= 🡪 ∠B = 70°, 110°

∠C = 180 – 50 – 70 = 60° or 180 – 50 – 110 = 20°

Largest area = x 18.4 x 15 x sin60 = 119.51°

AB2 = 18.42 + 152 – 2 x 18.4 x 15 x cos20 cos60 < cos20

AB = 6.70

Largest perimeter = 15 +18.4 + 6.70 = 40.10cm